



## New algorithms for quantum (symmetric) cryptanalysis

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# New Algorithms for Quantum (Symmetric) Cryptanalysis

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Joint work with André Chailloux<sup>2</sup> and Lorenzo Grassi<sup>1</sup>

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# Outline

- 1 Quantum-safe (Symmetric) Cryptography
- 2 Quantum Collision Search
- 3 Quantum k-xor Algorithms

# Quantum-safe (Symmetric) Cryptography

## (Pre-quantum) cryptography

Enable secure communications even in the presence of malicious adversaries.

### Asymmetric (e.g. RSA)

- No shared secret / computationally costly
- Security based on well-known hard mathematical problems (e.g. factorization)

### Symmetric (e.g. AES)

- Shared secret / computationally efficient
- Ideal security defined by generic attacks (e.g.  $2^{|K|}$ )
- Need of continuous security evaluation (cryptanalysis)

# A typical symmetric primitive

## Ideal block cipher

$E_K$  is a family of permutations of  $\{0, 1\}^n$  parameterized by  $K$ .

**Real** block cipher:

- Typically built by iterating a round function
- Select a key  $K$
- Decompose the message into  $n$ -bit blocks and use  $E_K$  with a **mode of operation**

# Generic attacks on ciphers

- The security provided by an **ideal block cipher** is defined by the best generic attack: exhaustive search for the key in  $2^{|K|}$
- Recovering the key from a secure cipher must be infeasible.

Typical key sizes range from  $|K| = 128$  to 256 bits.

# Symmetric cryptanalysis

- The ideal security is defined by generic attacks ( $2^{|K|}$ )
- Does real security meet this ideal security?
- We won't know ... without a continuous security evaluation.

Any attack better than the generic one is considered a “break”.

Cryptanalysis is an **empirical measure of security**.



# The security margin

The security of a cipher is not a 1-bit information:

- e.g. round-reduced attacks.

⇒ determine and adapt the **security margin**.

- The best attacks find the highest number of rounds reached (regardless of the complexity)
- Allows to compare primitives

# Quantum-safe (Symmetric) Cryptography

# Post-quantum cryptography

## Asymmetric (e.g. RSA)

- Shor's algorithm factorizes in polynomial time: this is not secure anymore.
- Actively looking for replacements (NIST call)

## Symmetric (e.g. AES)

Exhaustive search in  $2^{|K|/2}$  with Grover's algorithm.

- Double the key length for equivalent ideal security.

In both cases, lots of work regarding **quantum attacks**.

# Many new results

- Breaking some classically secure constructions in some quantum adversary models
- Extending cryptanalysis studies to quantum adversaries
- Solving recurrent generic problems

# Quantum search

Find in  $S$  (of size  $2^n$ ) an element  $x$  ( $2^t$  solutions) such that  $x$  satisfies some condition.

$$\underbrace{2^{(n-t)/2}}_{\substack{2^t \text{ solutions} \\ \text{among } 2^n}} \left( \underbrace{\text{Sampling}}_{\substack{\text{Produce the} \\ \text{search space } S \text{ in} \\ \text{superposition}}} + \underbrace{\text{Checking}}_{\substack{\text{Test a} \\ \text{superposition of} \\ x \in S}} \right)$$

## Two settings



### “Low-qubits”

Only  $\mathcal{O}(n)$  qubits, no qRAM access.

⇒ A quantum adversary from tomorrow.

### Exponential qRAM

Read and write access in quantum superposition:

$$\sum_i |i\rangle |0\rangle \rightarrow \sum_i |i\rangle |a_i\rangle$$

# Quantum Collision Search

*with A. Chailloux, M. Naya-Plasencia*

# The birthday problem

## Collision search

Let  $H : \{0, 1\}^n \rightarrow \{0, 1\}^n$  be a random function, find a collision of  $H$ , i.e. a pair  $x_1, x_2 \in \{0, 1\}^n$  such that  $H(x_1) = H(x_2)$ .

Numerous applications, e.g. generic attacks on hash functions.

- Classical time and queries:  $\Theta(2^{n/2})$
- With  $2^{n/2}$  queries, we can form  $2^n$  pairs, an  $n$ -bit collision occurs w.h.p.
- We can do this in  $\mathcal{O}(n)$  memory (Pollard's rho)



# Quantum algorithms for collisions

	Time	Queries	Qubits / qRAM	Classical memory
Pollard	$2^{n/2}$	$2^{n/2}$	0	$\mathcal{O}(n)$
Grover	$2^{n/2}$	$2^{n/2}$	$\mathcal{O}(n)$	0
Brassard, Høyer, Tapp	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$
BHT (*)	$2^{2n/3}$	$2^{n/3}$	$\mathcal{O}(n)$	$2^{n/3}$

# Collision search in a low-qubits setting



- Single-processor
- Only  $\mathcal{O}(n)$  qubits
- No qRAM lookups

# A naive collision algorithm

- Perform  $\ell$  arbitrary classical queries to  $H$ :  
 $H(x_1), \dots, H(x_\ell)$ .
- Search  $x \in \{0, 1\}^n$  such that:

$$H(x) \in \{H(x_1), \dots, H(x_\ell)\}$$

Optimal  $\ell = 2^{n/2}$ :

$$2^{n/2} + \frac{2^n}{2^{n/2}}$$



# A quantum collision algorithm

## Naive classical:

- Perform  $\ell$  arbitrary classical queries to  $H$ :  $H(x_1), \dots, H(x_\ell)$ .
- Search  $x \in \{0, 1\}^n$  such that:

$$H(x) \in \{H(x_1), \dots, H(x_\ell)\}$$

Optimal  $\ell = 2^{n/2}$ :

$$2^{n/2} + \frac{2^n}{2^{n/2}}$$

## Quantum (BHT):

- Perform  $\ell$  arbitrary classical queries to  $H$ :  $H(x_1), \dots, H(x_\ell)$ .
- **With Grover**, search  $x \in \{0, 1\}^n$  such that  $H(x) \in \{H(x_1), \dots, H(x_\ell)\}$ .

Optimal  $\ell = 2^{n/3}$ :

$$\underbrace{2^{\frac{n}{3}}}_{\text{List}} + \underbrace{\sqrt{\frac{2^n}{2^{n/3}}}}_{\text{Iterations}} \left( 1 + \underbrace{1}_{\text{qRAM lookup}} \right)$$



# Removing qRAM

We have a list  $L = \{H(x_1), \dots, H(x_\ell)\}$ , known classically, and want to compute:

$$|y\rangle |0\rangle \mapsto |y\rangle |y \in L\rangle .$$

- With qRAM: build a data structure for  $L$ , compute membership in  $\mathcal{O}(\log \ell)$  qRAM gates;
- Without qRAM: compare sequentially against elements of  $L$ .

We compute:

$$|y\rangle |0\rangle \mapsto |y\rangle |(y = H(x_1)) \vee (y = H(x_2)) \dots \vee (y = H(x_\ell))\rangle$$

in time  $\tilde{\mathcal{O}}(\ell)$ .

# BHT without quantum memory



Queries:

$$2^{n/3} + \sqrt{2^n / 2^{n/3}} (1 + 0)$$

Time:

$$2^{n/3} + 2^{n/3} (1 + 2^{n/3})$$



## Can we improve this?

Let's build a list of **distinguished points**, e.g.  $H(x_i) = 0^u || z$  for  $z \in \{0, 1\}^{n-u}$ .

- Building the list costs more:  $2^{n/3+u/2}$
- We have a setup cost (for searching among DPs):  $2^{u/2}$  per iteration
- The test still requires  $2^{n/3}$  time
- BUT less iterations:  $2^{n/3-u/2}$

$$\underbrace{\underbrace{2^{\frac{n}{3}}}_{\text{List size}} \times \underbrace{2^{\frac{u}{2}}}_{\text{Grover search of a DP}}}_{\text{First step: constructing the list}} + \underbrace{2^{\frac{n}{3}-\frac{u}{2}} \left( \underbrace{2^{\frac{u}{2}}}_{\text{Building all the DPs}} + \underbrace{2^{\frac{n}{3}}}_{\text{Lookup}} \right)}_{\text{Second step: searching a collision}}$$



## With optimal parameters

The cost becomes optimal for an intermediate list of size  $2^v \neq 2^{n/3}$ .

$$\underbrace{\underbrace{2^v}_{\text{List size}} \times \underbrace{2^{\frac{u}{2}}}_{\text{Grover search of a DP}}}_{\text{First step: constructing the list}} + \underbrace{\underbrace{2^{\frac{n-v-u}{2}}}_{\text{Less iterations}} \left( \underbrace{2^{\frac{u}{2}}}_{\text{Building all the DPs}} + \underbrace{2^v}_{\text{Lookup}} \right)}_{\text{Second step: searching a collision}}$$

With  $v = \frac{n}{5}$ ,  $u = \frac{2n}{5}$ , time:  $\tilde{O}(2^{2n/5})$ .

We also need  $2^{n/5}$  *classical memory*.



# Conclusion



- An asymptotic difference for collisions: time reduced from  $2^{n/2}$  to  $2^{2n/5}$
- Smallest number of computations when qRAM is not used
- More applications: multi-user settings, operation modes...

**Example:**  $n = 128$ ,  $2^{51}$  hash function queries instead of  $2^{64}$ , with less than 1GB classical data.

## State of the problem

	Time	Queries	Qubits	Classical memory
Pollard	$2^{n/2}$	$2^{n/2}$	0	$\mathcal{O}(n)$
Grover	$2^{n/2}$	$2^{n/2}$	$\mathcal{O}(n)$	0
BHT	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$
New	$2^{2n/5}$	$2^{2n/5}$	$\mathcal{O}(n)$	$2^{n/5}$

Can we meet the lower bound  $2^{n/3}$  with  $\mathcal{O}(n)$  qubits?

# Quantum k-xor Algorithms

*with L. Grassi, M. Naya-Plasencia (AC' 18)*

# Generalized Birthday Problem(s)

## Problem 1: The “original”

Given  $L_1, \dots, L_k$  **classical** lists of random  $n$ -bit strings, find  $x_1, \dots, x_k \in L_1 \times \dots \times L_k$  such that  $x_1 \oplus \dots \oplus x_k = 0$ .

## Problem 2: The “oracle”

Given **oracle access** to a random  $n$ -bit to  $n$ -bit function  $H$ , find  $x_1, \dots, x_k$  such that  $H(x_1) \oplus \dots \oplus H(x_k) = 0$ .

## Problem 3: The “unique solution”

Given **oracle access** to a random  $n/k$ -bit to  $n$ -bit function  $H$ , find the single  $k$ -tuple  $x_1, \dots, x_k$  such that  $H(x_1) \oplus H(x_2) \oplus \dots \oplus H(x_k) = 0$ .

## Focus on Problem 2 (with oracle)

### Problem 2: The “oracle” k-xor

Let  $H : \{0, 1\}^n \rightarrow \{0, 1\}^n$  be a random function, find  $x_1, \dots, x_k$  such that  $H(x_1) \oplus \dots \oplus H(x_k) = 0$ .

- Cryptanalysis: (R)FSB, SWIFFT...
- Applications for  $\oplus$  (bitwise XOR) and modular  $+$
- Related: approximate variants, subset-sums, decoding random linear codes, lattice problems...

# Examples

We note  $\tilde{O}(2^{\alpha_k n})$  the best time complexity of k-xor.

## The 1-xor Problem: exhaustive search

Searching  $x$  such that  $H(x) = 0$ : a preimage of 0. Simply use Grover's algorithm:  $\alpha_1 = 1/2$ .

## The 2-xor Problem: collision search

Previously:  $\alpha_2 = 1/3$  with qRAM and  $2/5$  without.

The problem becomes easier when  $k$  increases:  $\alpha_k$  is a decreasing function of  $k$ .

## Classical results for general k

To get a k-xor on  $n$  bits:

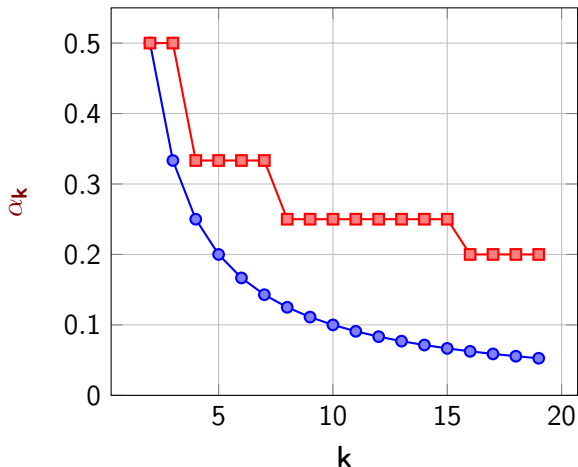
- The optimal **query complexity** is  $\Theta(2^{n/k})$
- The **time complexity** is  $\mathcal{O}(2^{n/(1+\lfloor \log_2(k) \rfloor)})$  (Wagner, 2002):

$$\alpha_k = \frac{1}{1+\lfloor \log_2(k) \rfloor}$$

- Logarithmic improvements in time
- We focus on **exponents**

# Classical results

$\alpha_k$  depending on  $k$



—●— Optimal queries  
 —■— Best time

The complexities  
 are  $\tilde{O}(2^{\alpha_k n})$



## Wagner's algorithm in a single slide

Let  $L_1$  and  $L_2$  be lists of  $2^u$  random values of  $H$ . Build  $L$ : among all pairs  $x_1, x_2 \in L_1 \times L_2$ , we take the partial collisions on the first  $u$  bits.

Then:

- $L$  contains  $2^u$  elements (there are  $2^{2u}$  pairs and a  $u$ -bit condition)
- $L$  can be built in time  $2^u$  if  $L_1$  and  $L_2$  are sorted

This works recursively: from two lists  $L_1, L_2$  of partial  $k$ -xors, we can obtain a list of  $2k$ -xors on more bits in time:

$$\text{MAX (size of the output list, MIN (size of } L_1, \text{size of } L_2))$$

## An example with $k = 4$

1. Query 4 lists of  $2^{n/3}$  single elements (values of  $H$ ): time  $2^{n/3}$

List of  $2^{n/3}$   
elements

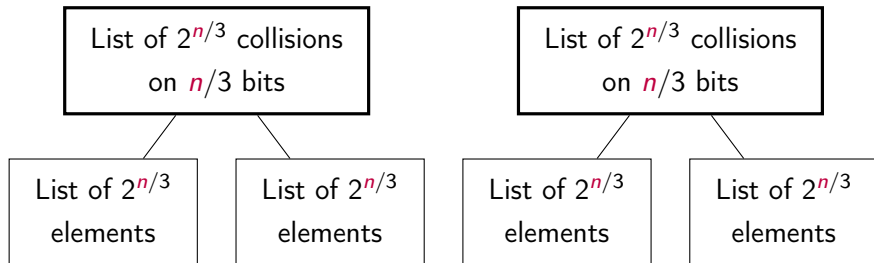
List of  $2^{n/3}$   
elements

List of  $2^{n/3}$   
elements

List of  $2^{n/3}$   
elements

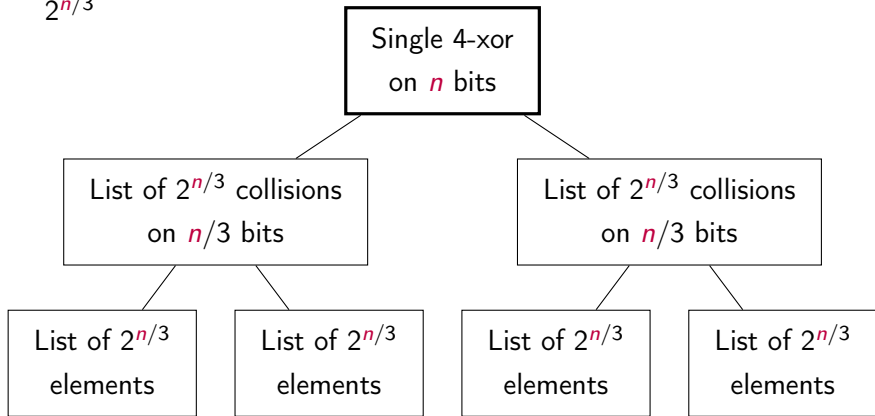
## An example with $k = 4$

1. Query 4 lists of  $2^{n/3}$  single elements (values of  $H$ ): time  $2^{n/3}$
2. Merge into two lists of  $2^{n/3}$  collisions on  $n/3$  bits: time  $2^{n/3}$



## An example with $k = 4$

1. Query 4 lists of  $2^{n/3}$  single elements (values of  $H$ ): time  $2^{n/3}$
2. Merge into two lists of  $2^{n/3}$  collisions on  $n/3$  bits: time  $2^{n/3}$
3. Find a collision between these lists: a single 4-xor of  $H$ : time  $2^{n/3}$

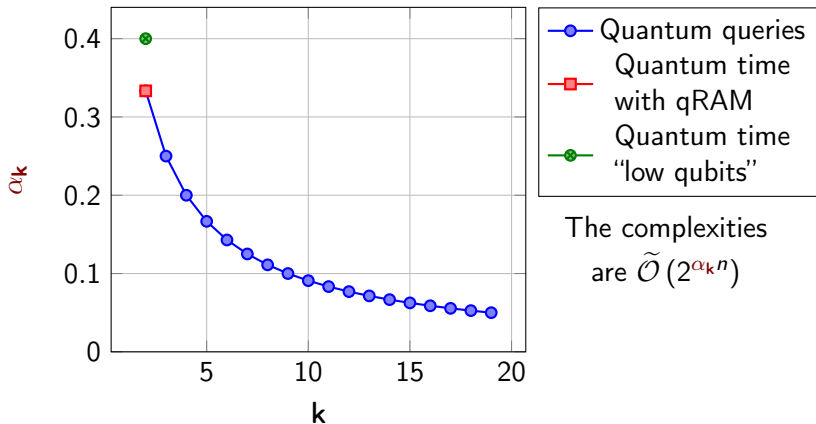


## Previous quantum results on k-xor

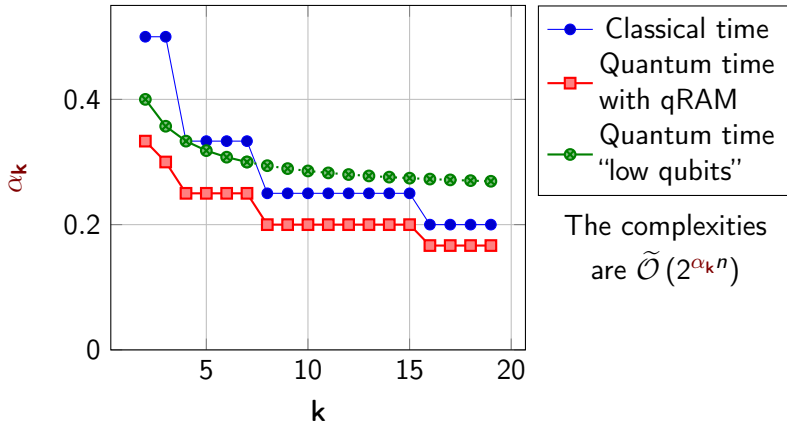
To get a k-xor on  $n$  bits:

- The optimal query complexity is  $\Theta(2^{n/(k+1)})$  (Belovs and Spalek)
- We know what happens for  $k = 2$ .
- For  $k > 2$ ?

## Previous quantum results



## Results of AC' 18



## Low-qubits merging strategy for $k = 3$

We don't have a single intermediate list, but two of them  $\Rightarrow$  they can be smaller.

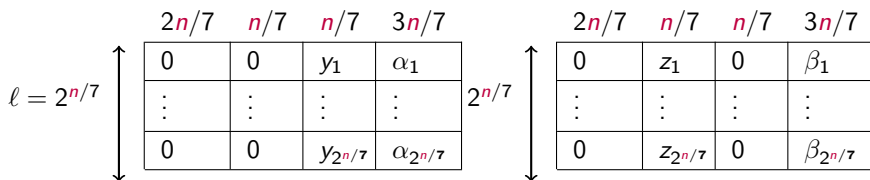
$$2^{n/8} \begin{array}{|c|c|} \hline \textcolor{red}{n/2} & \textcolor{red}{n/2} \\ \hline 0 & \alpha_1 \\ \hline \vdots & \vdots \\ \hline 0 & \alpha_{2^{n/8}} \\ \hline \end{array}$$

$$2^{n/8} \begin{array}{|c|c|} \hline \textcolor{red}{n/2} & \textcolor{red}{n/2} \\ \hline 0 & \beta_1 \\ \hline \vdots & \vdots \\ \hline 0 & \beta_{2^{n/8}} \\ \hline \end{array}$$

- Searching for a “distinguished solution”: we compare against all  $y, z \in L_1 \times L_2$
- Producing the lists costs  $2^{n/4} \times 2^{n/8} = 2^{3n/8}$  time and as much for searching  $x$ .



## Low-qubits merging strategy for $k = 3$ (ctd.)



We take more specific  $L_1$  and  $L_2$ . Checking a distinguished point  $x$ :

- **Match**  $L_1$  (find a partially colliding element); **then** match  $L_2$ ;
- Compute the xor of the three values.

$$2^{n/7+3^{n/14}} + \underbrace{2^{3^{n/14}}}_{\substack{3^{n/7} \\ \text{remaining} \\ \text{bits}}} \left( \underbrace{2^{n/7}}_{\text{Setup search space}} + \underbrace{\left( \underbrace{2^{n/7}}_{\text{Match } L_1} + \underbrace{2^{n/7}}_{\text{Match } L_2} \right)}_{\text{Instead of } 2^{n/7} \times 2^{n/7}} \right) = 2^{5^{n/14}}$$



## qRAM merging strategy for $k = 3$

$$\ell = 2^{n/5} \begin{array}{c} \updownarrow \\ \begin{array}{ccc} n/5 & n/5 & 3n/5 \\ \hline 0 & y_1 & \alpha_1 \\ \vdots & \vdots & \vdots \\ 0 & y_{2^{n/5}} & \alpha_{2^{n/5}} \end{array} \end{array}$$

$$2^{n/5} \begin{array}{c} \updownarrow \\ \begin{array}{ccc} n/5 & n/5 & 3n/5 \\ \hline z_1 & 0 & \beta_1 \\ \vdots & \vdots & \vdots \\ z_{2^{n/5}} & 0 & \beta_{2^{n/5}} \end{array} \end{array}$$

$$2^{n/5+n/10} + \underbrace{2^{3n/10}}_{3n/5 \text{ bits remaining}} \left( \underbrace{1}_{\text{Matching } L_1} + \underbrace{1}_{\text{Matching } L_2} \right) = 2^{3n/10} < 2^{n/3}$$

$\Rightarrow$  quantum 3-xor is exponentially faster than quantum collision search.

## Conclusion of AC' 18

Quantum 3-xor is exponentially faster than quantum collision search.

Low-qubits k-xor improves over classical for  $k \leq 7$ .

k-xor with qRAM in time  $\tilde{O}(2^{n/(2+\lfloor \log_2(k) \rfloor)})$  (instead of  $O(2^{n/(1+\lfloor \log_2(k) \rfloor)})$ ).

### Open questions

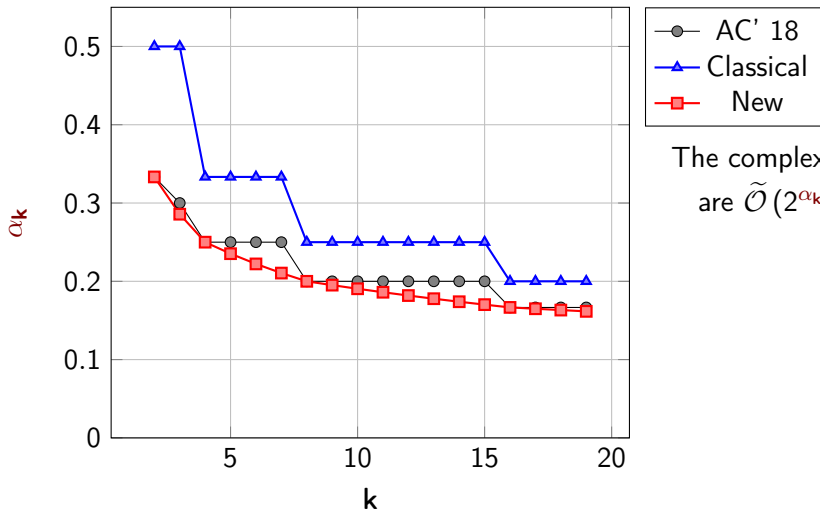
- A low-qubits speedup for all  $k$ ?
- With qRAM, other improvements than  $k = 3$ ?

# (Very) Recent Quantum Algorithms for k-xor

*with María Naya-Plasencia*



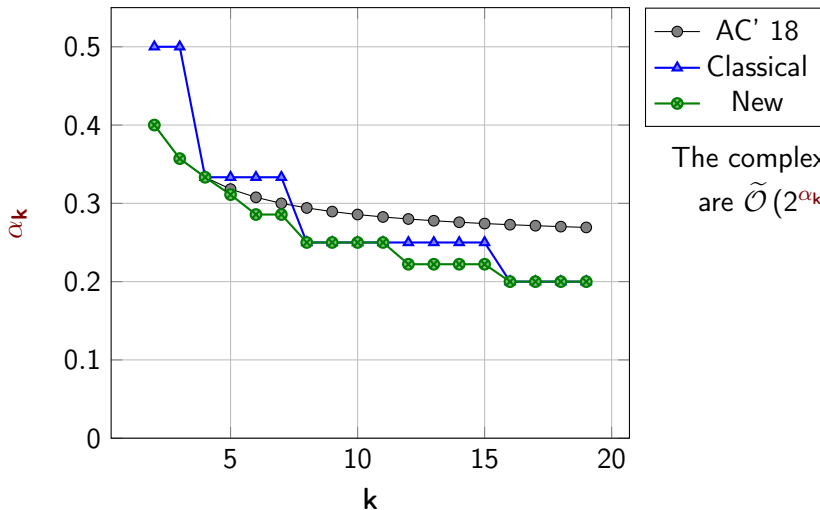
## Recent results (with qRAM)



The complexities  
are  $\tilde{O}(2^{\alpha_k n})$



## Recent results (low-qubits)



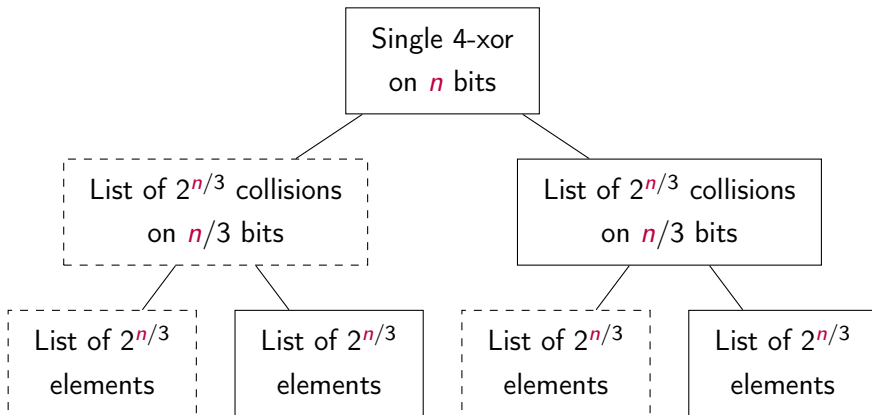
# History

- We found some punctual improvements, for some values of  $k$ ;
- We realized that all the possibilities could be included in a single framework: merging in a quantum-compliant way;
- We implemented an automatic search for the best merging strategies.

Merging strategies: build successive lists of partial  $\ell$ -xor for increasing  $\ell$ .

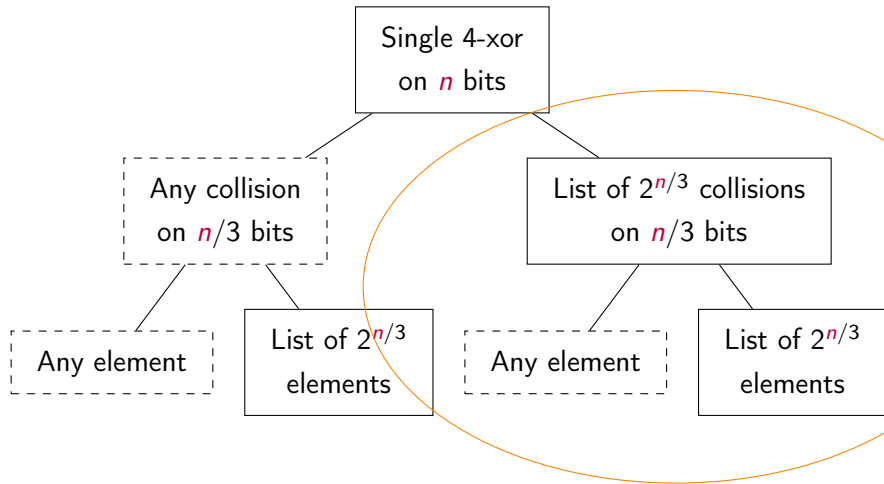
## Back to classical merging

Traverse the tree of merges in a depth-first manner (Wagner, 2002): store  $\lceil \log_2 k \rceil$  lists instead of  $k$ .

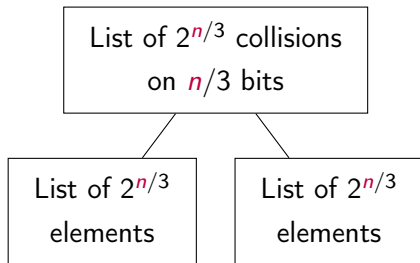




## Rephrasing the classical 4-xor algorithm



## From merging to matching



Before:

---

Two lists of  $2^{n/3}$  elements  
(random queries to  $H$ )



$2^{2n/3}$  pairs

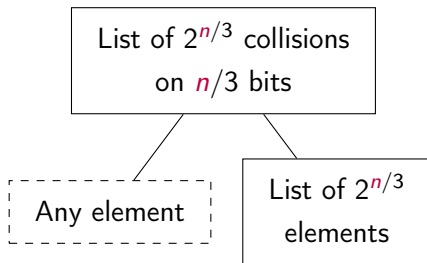


$2^{n/3}$  pairs with  $n/3$ -bit collision

---

In time  $2^{n/3}$  (sorted lists).

## From merging to matching



**After:**

---

A single list of  $2^{n/3}$  elements



Query  $H$  on the fly



Each query yields  $2^{n/3}$  pairs



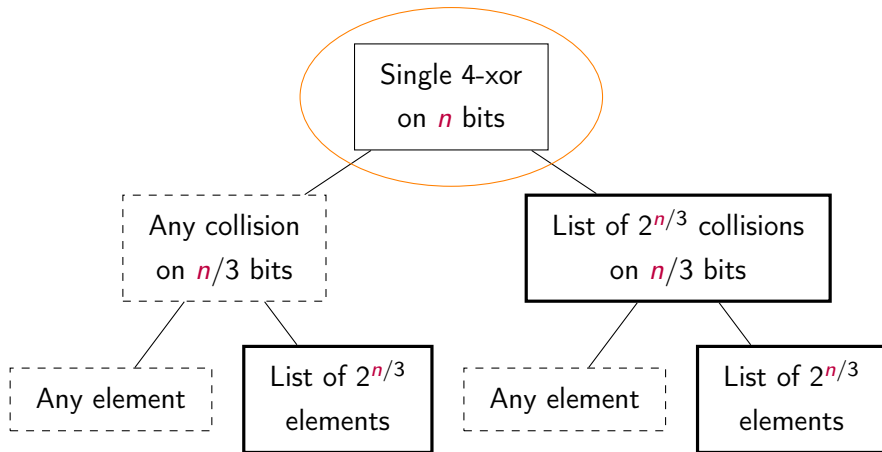
An  $n/3$ -bit collision

---

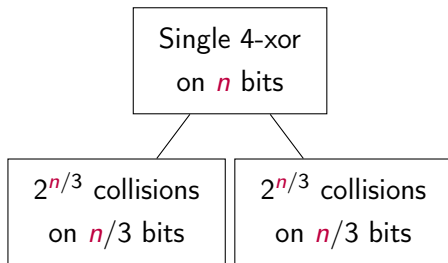
In time  $2^{n/3}$  (sorted list).

## From merging to matching (ctd.)

In this tree, each explicit list is built in time  $2^{n/3}$ .



## Merging at the root



Before:

---

Two lists of  $n/3$ -bit collisions



$2^{2n/3}$   $n/3$ -bit 4-xors

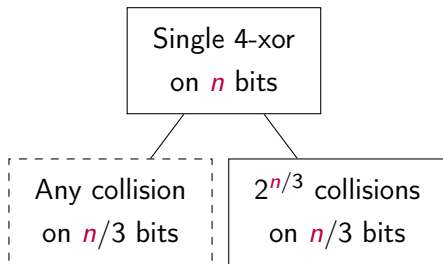


One  $n$ -bit 4-xor

---

In time  $2^{n/3}$  (sorted lists).

## Merging at the root



**After:**

---

A single list of  $n/3$ -bit collisions



Produce  $n/3$ -bit collisions on the fly



Each yields  $2^{n/3}$  4-tuples

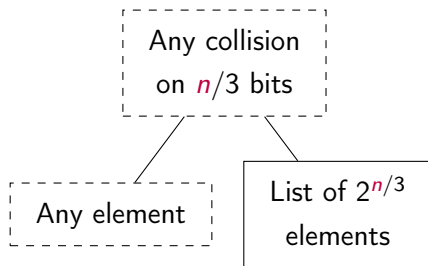


After  $2^{n/3}$  trials, a  $n$ -bit 4-xor

---

In time  $2^{n/3}$  (sorted list).

## Partial collisions on the fly



---

A single list of  $2^{n/3}$  elements



Query  $H$  on the fly



Each query yields  $2^{n/3}$  pairs



An  $n/3$ -bit collision

---

In time 1 (sorted list).

## In this example



- Explicit (intermediate) lists are built in time  $2^{n/3}$
- The last 4-xor is built by trying  $2^{n/3}$  partial collisions
- ...or trying  $2^{n/3}$  elements



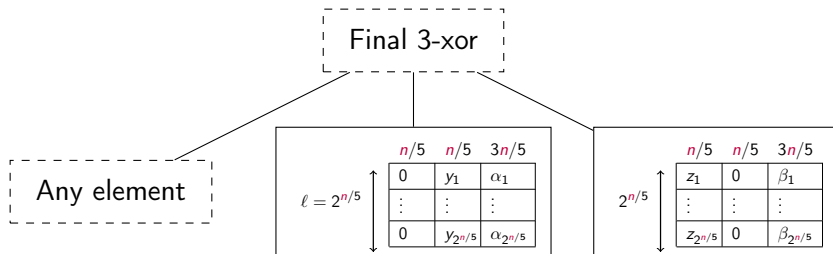
## In this example



- Explicit (intermediate) lists are built in time  $2^{n/3}$
- The last 4-xor is built by trying  $2^{n/3}$  partial collisions
- ...or trying  $2^{n/3}$  elements
- We can use Grover search in the last step: time  $2^{n/6}$
- ...we should balance the tree: at total time  $2^{n/4}$  in this example

# Rephrasing previous algorithms

The 3-xor algorithms with two intermediate lists: trees of height 2.



- We found a better merging for 3-xor with qRAM:  $\alpha_3 = \frac{2}{7} < \frac{3}{10}$
- (The low-qubits variant was optimal)

# Finding the best trees: MILP

We fix the tree structure.

- Variables: sizes of the lists, their costs (in  $\log_2$ ), prefixes
- Linear relations and constraints:
  - How we merge
  - How much this costs (classically or quantumly)
- An overall time complexity to minimize

## Theorem – with qRAM



### Theorem

If  $k \geq 2$  and  $\kappa = \lfloor \log_2(k) \rfloor$ , the best merging-tree quantum time exponent is

$$\alpha_k = \frac{2^\kappa}{(1 + \kappa)2^\kappa + k} .$$

Many trees give this time complexity, but one is obtained by using an “almost” binary tree.

## Theorem – qRAM-free



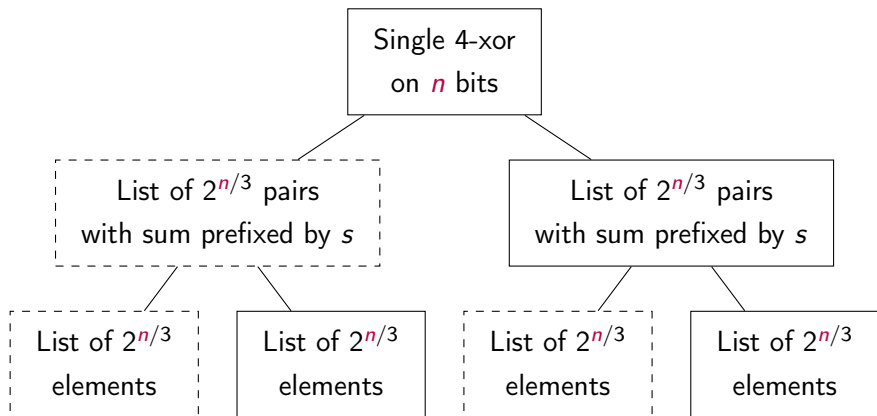
### Theorem

If  $k > 2$ ,  $k \neq 3, 5$  and  $\kappa = \lfloor \log_2(k) \rfloor$ , the best merging-tree quantum time exponent is:

$$\alpha_k = \frac{1}{\kappa+1} \text{ if } k < 2^\kappa + 2^{\kappa-1} \text{ or } \alpha_k = \frac{2}{2\kappa+3} \text{ if } k \geq 2^\kappa + 2^{\kappa-1}$$

Many trees give this time complexity, but one is obtained by using an “almost” binary tree.

## Extending the merging framework



If the search space is too small, loop over the values of the prefix  $s$ .



## Single-solution k-xor (Problem 3)

Given  $k$  lists of uniformly distributed  $n$ -bit strings, of size  $2^{n/k}$  each, find a  $k$ -xor on  $n$  bits.

- Previous work (Bernstein, Jeffery, Lange, Meurer, 2013): if  $k$  is a multiple of 4, time  $\tilde{O}(2^{0.3n})$  with a quantum walk.
- New: quantum time  $\tilde{O}(2^{\beta_k n})$  with  $\beta_k = \frac{1}{k} \frac{k + \lceil k/5 \rceil}{4}$ , without a quantum walk.
  - Improves all  $k$  except multiples of 4
  - Meets 0.3 when  $k$  is a multiple of 5
  - Applies to  $k$ -encryption

# Conclusion

- We have found the optimal merging trees for quantum k-xor
- All of this works when replacing  $\oplus$  by  $+$
- We extended this to problems with less solutions and without quantum oracle access (Problem 1)



## Future work / open questions

- Extend the framework (more techniques)
- Extend the cryptographic applications (approximate problems)

### Open questions

- Quantum time complexity of collision search with  $\mathcal{O}(n)$  qubits (“why  $2/5$ ?”)
- Quantum time complexity of k-xor with a single solution (“why  $0.3$ ?”)

Thank you.